Lesson 4: Solving Quadratic Equations by the Quadratic Formula

Standards Addressed:

- **M9AL-Ia-b-1** - solves quadratic equations by: (a) extracting square roots; (b) factoring; (c) completing the square; and (d) **using the quadratic formula**.

In the past lessons, we learned how to solve quadratic equations by extracting square roots, factoring, and completing the square. There is one last way to solve quadratic equations, and it is by using the quadratic formula. The formula is as follows:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Remember that a standard quadratic equation is in the form $ax^2 + bx + c = 0$. To use the quadratic equation, you would just input the values for a, b, and c to find the roots of the equation. This is best shown with examples.

Example 1

Solve the equation $x^2 - 6x + 5 = 0$.

$x^2 - 6x + 5 = 0$		$a = 1, b = -6, c = 5$
$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)}$		Use quadratic formula.
$x = \frac{6 \pm \sqrt{36 - 20}}{2}$		Simplify.
$x = \frac{6 \pm \sqrt{16}}{2}$		
$x = \frac{6 \pm 4}{2}$		$\sqrt{16} = 4$
$x = \frac{6 + 4}{2}$	$x = \frac{6 - 4}{2}$	Separate each roots.
$x = \frac{10}{2}$	$x = \frac{2}{2}$	Simplify.
$x = 5$	$x = 1$	

Here are the steps on using the quadratic formula.

1. Transfer all terms on the left side so that the right side equates to 0.
2. List the numerical values of the coefficients a, b, and c.
3. Plug in the values into the quadratic formula.
4. Solve for x using the quadratic formula.
5. (Optional) Plug in the roots into the quadatic formula to see if the answer is correct.

Example 2

Solve the equation $x^2 + 8x = 16$.

$x^2 + 8x = 16$	
$x^2 + 8x - 16 = 0$	Subtract 16 on both sides.
$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-16)}}{2(1)}$	$a = 1, b = 8, c = -16$
$x = \frac{-8 \pm \sqrt{64 + 64}}{2}$	Simplify.
$x = \frac{-8 \pm 8\sqrt{2}}{2}$	
$x = -4 \pm 4\sqrt{2}$	Simplify fraction.

Checking:

Solution 1	
$\left(-4 + 4\sqrt{2}\right)^2 + 8\left(-4 + 4\sqrt{2}\right) = 16$	Substitute $-4 + 4\sqrt{2}$.
$48 - 32\sqrt{2} - 32 + 32\sqrt{2} = 16$	Simplify.
$48 - 32 = 16$	Combine like terms.
$16 = 16$	
Solution 2	
$\left(-4 - 4\sqrt{2}\right)^2 + 8\left(-4 - 4\sqrt{2}\right) = 16$	Substitute $-4 - 4\sqrt{2}$.
$48 + 32\sqrt{2} - 32 - 32\sqrt{2} = 16$	Simplify.
$48 - 32 = 16$	Combine like terms.
$16 = 16$	

Example 4

Solve for the equation $x^2 + x - 2 = 0$.

$x^2 + x - 2 = 0$		
$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-2)}}{2(1)}$		$a = 1, b = 1, c = -2$
$x = \frac{-1 \pm \sqrt{1 + 8}}{2}$		Simplify.
$x = \frac{-1 \pm \sqrt{9}}{2}$		
$x = \frac{-1 \pm 3}{2}$		$\sqrt{9} = 3$
$x = \frac{-1 + 3}{2}$	$x = \frac{-1 - 4}{2}$	Separate each roots.
$x = \frac{2}{2}$	$x = -\frac{5}{2}$	
$x = 1$	$x = -\frac{5}{2}$	

Example 5

Solve for the equation $4x^2 + \frac{7x}{6} - \frac{5}{3} = 0$.

You could input the coefficients immediately into the formula, but you could make the process later easier by multiplying the LCD of the fractions so that the coefficients will be whole numbers.

$4x^2 + \frac{7x}{6} - \frac{5}{3} = 0$	
$6\left(4x^2 + \frac{7x}{6} - \frac{5}{3}\right) = 6(0)$	Multiply both sides by 6.
$24x^2 + 7x - 10 = 0$	
$x = \frac{-7 \pm \sqrt{7^2 - 4(24)(-10)}}{2(24)}$	$a = 24, b = 7, c = -10$
$x = \frac{-7 \pm \sqrt{49 + 960}}{48}$	Simplify.
$x = \frac{-7 \pm \sqrt{1009}}{48}$	

Since there are no more simplifications to be made on $\sqrt{1009}$, you could leave your answer at that or write your answer as $x = \frac{-7+\sqrt{1009}}{48}$ and $x = \frac{-7-\sqrt{1009}}{48}$. Both answers are acceptable.

Practice

Do the following activity on Khan Academy:

1. Quadratic Formula